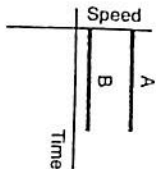
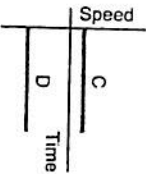


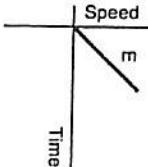
Accelerated motion of various kinds along straight paths can be shown on speed-time graphs (see Figure 1-3). For example, a horizontal line shows constant speed (no acceleration); a line with positive slope shows increasing speed (acceleration); and one with negative slope shows decreasing speed (negative acceleration or deceleration). A line that crosses the time axis is interpreted as a change of direction: the speed in one direction decreases to zero at the time the graph line crosses the time axis, and then the speed increases in the opposite direction. A curved speed-time line indicates that the acceleration is not constant.



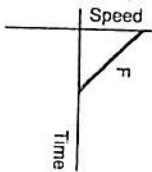
(a) Both A and B are moving at constant speed; acceleration is zero. A is traveling faster than B.



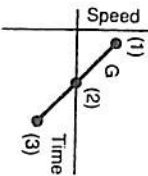
(b) C and D have zero acceleration, constant speed. D is moving faster than C, but in the opposite direction.



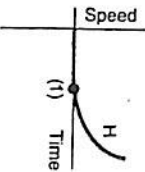
(c) E is accelerating uniformly.



(d) The speed of F is decreasing uniformly.



(e) G is decreasing speed from (1) to (2), is motionless at (2), and increases speed in the opposite direction from (2) to (3). Slope and acceleration are negative at all times. (An object thrown upward would have this curve.)

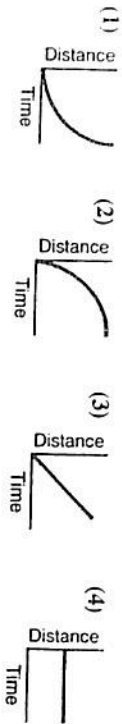


(f) H is at rest up to time (1), and has nonuniform acceleration after that time.

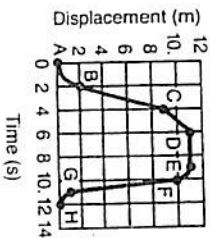
Figure 1-3. Graphs of various types of motion in a straight path, drawn on speed-time axes.

QUESTIONS

1. Which graph represents the motion of an object moving with a constant speed?



2. Which is a vector quantity? (1) speed (2) time (3) velocity (4) distance
- 3-7. Base your answers to Questions 3 through 7 on the graph at right, which represents the motion of a cart traveling for 12 seconds along a straight line.



3. The cart was at rest during the interval (1) AB (2) BC (3) DE (4) GH
4. The total distance covered by the cart in 12 seconds was (1) 6 m (2) 11 m (3) 12 m (4) 22 m
5. The average speed of the cart during interval CD was (1) 1 m/s (2) 2 m/s (3) 10. m/s (4) 11 m/s
6. An interval during which the cart was moving with constant velocity is (1) AB (2) BC (3) EF (4) GH
7. A part of the trip during which the cart's velocity was not constant is represented by the line (1) AB (2) BC (3) DE (4) FG

Final Velocity and Distance Traveled During Constant Acceleration

The average speed, \bar{v} , of a body that accelerates uniformly from an initial speed, v_i , to a final speed, v_f , can be written as

$$\bar{v} = \frac{v_f + v_i}{2} \quad (\text{Eq. 1-3})$$

By rearranging Equation 1-2 (page 7), you can see that $\Delta v = a\Delta t$. Into this formula you can substitute $v_f - v_i$ for Δv , which gives

$$v_f - v_i = a\Delta t$$

$$v_f = v_i + a\Delta t$$

Substituting this value of v_f into Equation 1-3 gives

$$\bar{v} = \frac{(v_i + a\Delta t) + v_i}{2} = v_i + \frac{1}{2}a\Delta t$$

Substituting this formula for \bar{v} in Equation 1-1a

$$\Delta s = \bar{v}\Delta t \quad (\text{Eq. 1-1a})$$

gives

$$\Delta s = (v_i + \frac{1}{2}a\Delta t)\Delta t$$

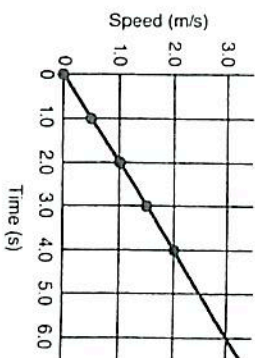
$$\Delta s = v_i\Delta t + \frac{1}{2}a(\Delta t)^2 \quad (\text{Eq. 1-4})$$

PRACTICAL APPLICATION

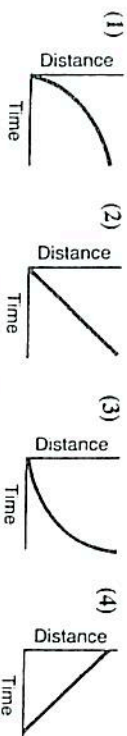
Examples of free fall include amusement park rides such as the roller coaster and sports such as sky diving (assuming negligible air resistance). ■

QUESTIONS

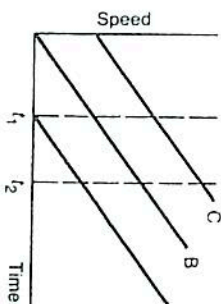
1. The graph at right shows the speed of an object plotted against time. The total distance traveled by the object during the first 4.0 seconds is (1) 0.50 m (2) 2.0 m (3) 8.0 m (4) 4.0 m



2. How far will a freely falling object, initially at rest, fall in 2.4 seconds?
(1) 6.0 m (2) 12 m (3) 23 m (4) 28 m
3. An object originally at rest is uniformly accelerated along a straight-line path to a speed of 8.0 meters/second in 2.00 seconds. The acceleration of the object is (1) 0.25 m/s² (2) 10. m/s² (3) 16 m/s² (4) 4.0 m/s²
4. The time-rate of change of displacement is (1) acceleration (2) distance (3) velocity (4) speed
5. A body falls freely from rest near the surface of the earth. The distance the body falls in 2.00 seconds is (1) 2.3 m (2) 7.1 m (3) 12 m (4) 20. m
6. Which graph represents the motion of a freely falling object near the surface of the earth?

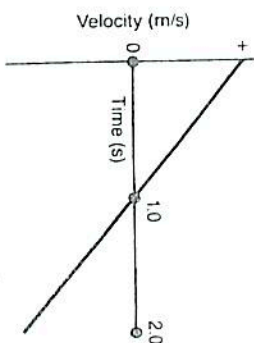


7. To change the velocity of an object, there *must* be (1) an increase in its speed (2) a decrease in its speed (3) a change in either its speed or direction (4) a change in both its speed and direction
8. The motions of cars A, B, and C in a straight path are represented by the graph at right. During the time interval from t_1 to t_2 , the three cars travel (1) the same distance (2) with the same speed only (3) with the same acceleration only (4) with both the same speed and the same acceleration

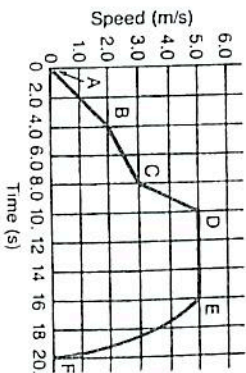


9. An object is accelerated uniformly from rest to a speed of 25 meters per second in 10. seconds. The acceleration of the object is (1) 1.0 m/s² (2) 2.5 m/s² (3) 1.5 m/s² (4) 2.5 m/s²

10. Starting from rest, how far can a 2.00-kilogram mass fall freely in 1.00 second? (1) 4.90 m (2) 2.00 m (3) 9.80 m (4) 19.6 m
11. As the time required to accelerate an object from rest to a speed of 4 meters per second decreases, the acceleration of the object (1) decreases (2) increases (3) remains the same (4) remains the same
12. As an object falls freely near the earth, its acceleration (1) decreases (2) increases (3) remains the same (4) remains the same

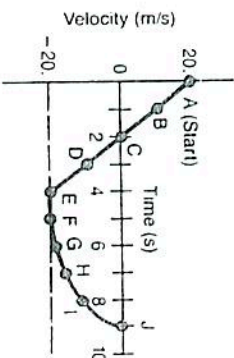


13. The velocity-time graph at right represents the motion of a ball released vertically upward at $t = 0$. During the time interval 1.0 second to 2.0 seconds, the displacement of the ball from the point where it was released (1) decreases (2) increases (3) remains the same (4) remains the same



- 14-18. Base your answers to Questions 14 through 18 on the graph at right, which represents the changing speed of a cart during an interval of 20. seconds.
14. What is the distance traveled by the cart during interval AB? (1) 1.0 m (2) 2.0 m (3) 8.0 m (4) 4.0 m
15. In which of the following intervals is the magnitude of the cart's acceleration greatest? (1) AB (2) BC (3) CD (4) DE
16. During interval BC, the magnitude of the cart's acceleration is (1) 1.0 m/s² (2) 0.25 m/s² (3) 0.50 m/s² (4) 4.0 m/s²
17. The average speed of the cart during interval CD is (1) 15 m/s (2) 2.0 m/s (3) 7.5 m/s (4) 4.0 m/s
18. The cart's acceleration is changing during interval (1) BC (2) CD (3) DE (4) EF

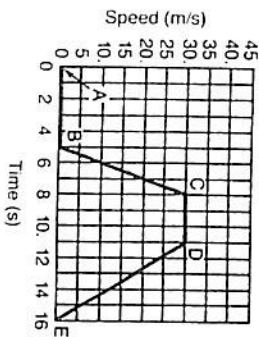
- 19-23. Base your answers to Questions 19 through 23 on the velocity-time graph at right.



19. During which interval is the magnitude of the object's acceleration greatest? (1) EF (2) FG (3) GH (4) IJ

20. The acceleration of the object at point *D* on the curve is (1) 0 m/s^2 (2) 5 m/s^2 (3) -10 m/s^2 (4) -20 m/s^2
21. During what interval does the object have zero acceleration? (1) *BC* (2) *EF* (3) *GH* (4) *HI*
22. At what point is the object's displacement from the start equal to zero? (1) *C* (2) *E* (3) *F* (4) *J*
23. At what point is the object's displacement from the start a minimum? (1) *C* (2) *E* (3) *G* (4) *J*

24–28. Base your answers to Questions 24 through 28 on the speed-time graph at right representing the motion of a cart.



24. The cart travels the shortest distance in interval (1) *AB* (2) *BC* (3) *CD* (4) *DE*
25. The cart's average speed is greatest during interval (1) *AB* (2) *BC* (3) *CD* (4) *DE*
26. The cart travels the greatest distance during interval (1) *AB* (2) *BC* (3) *CD* (4) *DE*
27. The magnitude of the cart's acceleration is greatest during interval (1) *AB* (2) *BC* (3) *CD* (4) *DE*
28. During how many intervals is the cart's acceleration equal to zero? (1) 1 (2) 2 (3) 3 (4) 4
29. A cart initially traveling at 10. meters per second north accelerates uniformly at $3.0 \text{ meters per second squared}$ to the north for 4.0 seconds. The displacement of the cart from its initial position at the end of this 4.0 seconds is (1) 40. m north (2) 64 m north (3) 88 m north (4) 180 m north
30. A cart initially traveling at 10. meters per second to the right and accelerating uniformly at $2.0 \text{ meters per second squared}$ to the right is displaced 11 meters. The final velocity of the cart is (1) 2.0 m/s right (2) 2.0 m/s left (3) 12 m/s right (4) 12 m/s left
31. An object initially traveling at 20. meters per second west decelerates uniformly at 4.0 meters per second squared for 2.0 seconds. The displacement of the object during these 2.0 seconds is (1) 32 m east (2) 32 m west (3) 48 m east (4) 48 m west
32. An object initially traveling at 20. meters per second south decelerates uniformly at $6.0 \text{ meters per second squared}$ and is displaced 25 meters. The final velocity of the object is (1) 26 m/s north (2) 26 m/s south (3) $10. \text{ m/s}$ north (4) $10. \text{ m/s}$ south

II. STATICS

The division of mechanics that studies the relation between forces acting on an object at rest is called statics.

Concurrent Forces

Any push or pull is a force. Force is a vector quantity because it is always associated with a direction. In the *SI* system, the magnitude of a push or pull is expressed in newtons, N, a derived unit. (Force and the derivation of the newton will be further explained in Section III, Dynamics.)

Two or more forces that act on the same body at the same time are called **concurrent forces**. The single force that is equivalent to the combined effect of these concurrent forces is called the **resultant**.

If two concurrent forces act in the same direction (at an angle of 0° to each other), their resultant is simply the sum of their magnitudes, acting in the same direction as the two forces. This is the largest resultant that two forces can have. If the two forces act in the same line, but in opposite directions (at an angle of 180° to each other), their resultant is the *difference* of their magnitudes, acting in the direction of the larger force. This is the smallest possible resultant of the two forces. Figure 1-6 illustrates these facts for a force of 8.0 N acting concurrently with a force of 6.0 N. The resultant for an angle of 0° is 14.0 N; for an angle of 180° , it is 2.0 N in the direction of the 8.0-N force.

(a) Vectors acting in the same direction

Adding the vectors (head-to-tail method)



(b) Vectors acting in opposite directions

Adding the vectors (head-to-tail method)

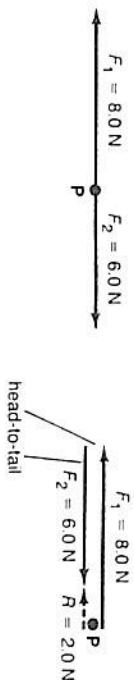


Figure 1-6. The resultants, *R*, of concurrent forces acting along the same straight line: (a) acting in the same direction, and (b) acting in opposite directions.

Triangle Method of Adding Concurrent Forces. The resultant of two concurrent forces acting at an angle between 0° and 180° can be found by the triangle method of vector addition. In this method, each force is represented by a vector arrow drawn to scale, with its length corresponding to the magnitude of the force and its direction corresponding to the direction of the force. To add the two vectors, the tail of the second vector is placed at the head of the first vector.

The first part of the paper discusses the general theory of the problem. It is shown that the problem is equivalent to a certain type of boundary value problem for a second order elliptic partial differential equation. The problem is then reduced to a problem of finding the solution of a certain type of integral equation.

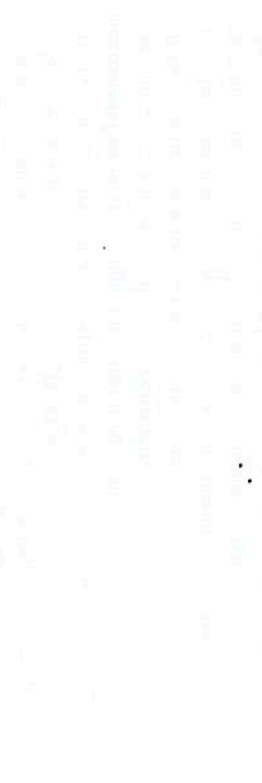


The numerical method is based on the use of a certain type of finite difference scheme. It is shown that the scheme is stable and convergent. The results of the numerical calculations are presented in a table.

The results of the numerical calculations are compared with the results of the analytical solution. It is shown that the numerical results are in good agreement with the analytical results. The error of the numerical method is estimated to be of the order of 10^{-4} .

The paper concludes with a discussion of the results and a list of references. The authors are grateful to the National Science Foundation for its support of this work.

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